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NORMAL STRESS EFFECTS IN VISCOELASTIC FLUID LUBRICATION

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Summary

Non-Newtonian flow effects are evaluated in a slider-bearing configuration. The material model taken is that of the Coleman—Noll second-order fluid. An explicit result is given for the portion of the bearing load supported by the non-Newtonian normal stresses as well as that portion resulting from the usual lubrication theory (Newtonian effect). Particular attention is given to the non-Newtonian effect of a high-polymer additive applied to a Newtonian base stock. The non-Newtonian effect has a particular dependence on the bearing geometry as well as a dependence on the relaxation time of the additive and the amount by which the additive increases the viscosity. The strength of the non-Newtonian effect is assessed in realistic conditions of bearing operation. We find that under certain conditions the non-Newtonian effect could provide a significant load-supporting capability. However, with slight changes in the conditions of the bearing operation, the non-Newtonian load support is negligible. These results are interpreted and qualified with respect to the limitations of the second-order theory, which does not include shear thinning effects.

1. Introduction

The use of polymeric additives in lubricating oils is widespread and has several advantages. The primary effect is to stabilize the flow properties of

the oils so they are as insensitive as possible to changes in temperature. A related effect concerns the use of additives to minimize the sensitivity of the lubricating oil to changes in shear rate. It is often claimed that oils with additives have greater load-bearing capacities in bearings, which results in longer bearing life. The mechanism of this presumed effect is not well understood. One possible mechanism, related to non-Newtonian behavior, is reported here.

It is necessary to be more specific about the meaning of the term "increased load capacity". By that we mean, the bearing can sustain a greater load in the presence of oil with the polymer additive than it can with a different oil of the same viscosity but without the additive. In other words, we are seeking an explanation for the increased load capacity of the bearing other than the effect the additive has upon the viscosity. The usual explanation lies in the "fluid-elasticity" effect commonly observed in oils with additives. The existence of an elasticity characteristic in the modified oil implies that the material must be characterized as being non-Newtonian, with the inherent effects of a relaxation-time spectrum and the generation of normal stresses in shearing flows.

By no means is it obvious that non-Newtonian effects enhance the capacity of a bearing to sustain load. The analysis by Brindley et al. [1] indicates that an additive would decrease the load capacity of a squeeze-film bearing. This is a contradiction to their experimental data and is discussed fully by them. It is apparent there is a need for further theoretical modeling. For the slider and journal bearings, there seems to be agreement that non-Newtonian elasticity effects could in principle enhance the load capacity, but it has been argued that at high rates, thinning effects render the non-Newtonian normal stress effect to be negligibly small as shown by Tanner [2]. However, there is no agreement on the magnitude and importance of certain separate mechanisms for non-Newtonian behavior. For example, Harnoy [3,4] argues that

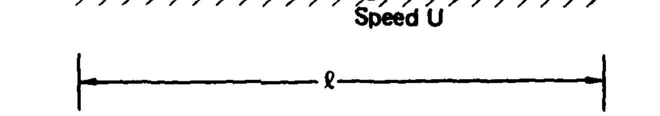

Fig. 1. Slider bearing.

the predominate response is of a transient type in the range of a characteristic relaxation time. However, to pursue his thesis, he finds it necessary to make the somewhat *ad hoc* introduction of two separate relaxation times, one associated with shear response and one associated with normal stress response. It is difficult to rationalize this assumption in terms of molecular mechanisms.

The approach followed here is to examine the normal stress effect. To be sure, this has been done before; however, we employ a different constitutive model than used previously. Specifically, our interest here is in the behavior of the slider-bearing configuration (see Fig. 1). This problem was solved by Tanner [5] using the second-order, Rivlin—Ericksen fluid characterization. Here we solve this problem at the same level of characterization, second-order theory, but using the Coleman—Noll fluid characterization. As will be seen, the advantage of the latter procedure is that the final results are expressed very simply in terms of viscosity coefficients and a relaxation time, with no additional undetermined material parameters.

It is in the nature of the second-order fluid characterization that the viscosity character of the fluid is independent of shear rate; thus shear thinning is not generally modeled by these truncated theories. To consider shear-thinning effects, we could consider the viscosity function to be shear-rate dependent, as in Hsu and Saibel [6]; however, we do not pursue that approach here. Even though we do not explicitly model the effects of shear stress thinning and normal stress thinning, we are interested in the *relative* magnitude of these effects, for which the model may provide some guidance. The significance of shear thinning effects also will be discussed later. We shall obtain an explicit result for the load carried by the slider bearing as a function of the fluid properties. We shall interpret the result in terms of the practical lubrication practice of using oils with high-polymer additives.

2. Analysis

The starting point of the analysis is the incompressible fluid characterization through the Coleman—Noll [7] second-order form, i.e.,

$$\begin{aligned}\sigma_{ij} = -p\delta_{ij} &+ \int_{-\infty}^{t} \Delta(t-\tau)\,\frac{\partial G_{ij}(\tau)}{\partial \tau}\,\mathrm{d}\tau \\ &+ \int_{-\infty}^{t}\int_{-\infty}^{t} \beta(t-\tau, t-\eta)\,\frac{\partial G_{ik}(\tau)}{\partial \tau}\,\frac{\partial G_{kj}(\eta)}{\partial \eta}\,\mathrm{d}\tau\,\mathrm{d}\eta \\ &- \int_{-\infty}^{t}\int_{-\infty}^{t} \gamma(t-\tau, t-\eta)\,\frac{\partial G_{ij}(\tau)}{\partial \tau}\,\frac{\partial G_{kk}(\eta)}{\partial \eta}\,\mathrm{d}\tau\,\mathrm{d}\eta\,,\end{aligned} \tag{1}$$

where

$$G_{ij}(\tau) = \frac{\partial x_k(\tau)}{\partial x_i(t)} \frac{\partial x_k(\tau)}{\partial x_j(t)} - \delta_{ij}, \tag{2}$$

with rectangular Cartesian coordinates being employed. The terms $\Delta(\ )$, $\beta(\ )$ and $\gamma(\ )$ are the relaxation functions of the theory. Using Coleman's thermodynamical method [8], Christensen [9] showed that the $\Delta(\ )$ and $\beta(\ )$ relaxation functions in (1) are not independent if the stress constitutive relation is to be derived from a free-energy functional. In fact, from the thermodynamical derivation, (1) takes the reduced form

$$\begin{aligned}\sigma_{ij} = -p\delta_{ij} &+ \int_{-\infty}^{t} \Delta(t-\tau, 0) \frac{\partial G_{ij}(\tau)}{\partial \tau} \, d\tau \\ &- \int_{-\infty}^{t} \int_{-\infty}^{t} \Delta(t-\tau, t-\eta) \frac{\partial G_{ik}(\tau)}{\partial \tau} \frac{\partial G_{kj}(\eta)}{\partial \eta} \, d\tau \, d\eta \\ &- \int_{-\infty}^{t} \int_{-\infty}^{t} \gamma(t-\tau, t-\eta) \frac{\partial G_{ij}(\tau)}{\partial \tau} \frac{\partial G_{kk}(\eta)}{\partial \eta} \, d\tau \, d\eta,\end{aligned} \tag{3}$$

where $\Delta(\ )$ and $\gamma(\ )$ are taken to be symmetric in their arguments.

The application to the slider bearing will involve the restriction of (3) to states of simple shearing flow. Let the shearing flow be specified by $v_1 = \kappa X_2$, $v_2 = 0$, and $v_3 = 0$. Then under steady-state conditions, (3) reduces to

$$\begin{aligned}\sigma_{11} &= -p - \kappa^2 \int_0^{\infty} \int_0^{\infty} \Delta(u, v) \, du \, dv, \\ \sigma_{22} &= -p - 2\kappa^2 \int_0^{\infty} \Delta(u, 0) \, u \, du - \kappa^2 \int_0^{\infty} \int_0^{\infty} \Delta(u, v) \, du \, dv \\ &\quad - 4\kappa^4 \int_0^{\infty} \int_0^{\infty} \Delta(u, v) \, uv \, du \, dv - 4\kappa^4 \int_0^{\infty} \int_0^{\infty} \gamma(u, v) \, uv \, du \, dv, \\ \sigma_{33} &= -p, \\ \sigma_{12} &= \kappa \int_0^{\infty} \Delta(u, 0) \, du + 2\kappa^3 \int_0^{\infty} \int_0^{\infty} \Delta(u, v) \, v \, du \, dv \\ &\quad + \kappa^3 \int_0^{\infty} \int_0^{\infty} \gamma(u, v) \, v \, du \, dv, \\ \sigma_{23} &= \sigma_{31} = 0.\end{aligned} \tag{4}$$

Finally, restricting all terms to only lowest-order effects gives

$$\sigma_{11} = -p - \kappa^2 \int_0^\infty \int_0^\infty \Delta(u, v)\, \mathrm{d}u\, \mathrm{d}v,$$

$$\sigma_{22} = -p - 2\kappa^2 \int_0^\infty \Delta(u, 0)\, u\, \mathrm{d}u - \kappa^2 \int_0^\infty \int_0^\infty \Delta(u, v)\, \mathrm{d}u\, \mathrm{d}v, \tag{5}$$

$$\sigma_{12} = \kappa \int_0^\infty \Delta(u, 0)\, \mathrm{d}u.$$

The simplicity of (5), involving only a single relaxation function $\Delta(u, v)$, will be of great utility to us later. It should be emphasized that the reduction of the Coleman—Noll second-order theory form results from the thermodynamical restriction that (5) be derived from a free-energy functional. Note from (5) that, in terms of the normal stress differences $N_1 = \sigma_{11} - \sigma_{22}$ $N_2 = \sigma_{22} - \sigma_{33}$, there results $|N_2| > N_1$, which is at variance with much experimental evidence. However, because our purpose is merely to assess the magnitude of normal stress effects, the form (5) has the overwhelming advantage of involving only a single mechanical property $\Delta(u, v)$, as well as being derived in a rational theoretical framework. Also, there is evidence that in approaching a low-rate asymptotic limit, $|N_2|$ and N_1 are of the same size, and the smallness generally attributed to $|N_2|$ compared with N_1 results from thinning effects, such as in shear thinning with high rate. This matter is discussed at some length in Ref. 10. The main point is that the forms of (5) result from a consistent theoretical derivation, but we employ them with caution outside the range of their strict validity, as is common practice. Because we are only looking for the relative size of normal stress to shear stress, rather than absolute values of each, the application of (5) at high rates of deformation may be justifiable on an order-of-magnitude basis over a certain range of rates.

Next, we characterize the relaxation function that appears in (5). Specifically, we take

$$\Delta(t) = \beta_0\, \mathrm{e}^{-t/\tau_0} + \beta_a\, \mathrm{e}^{-t/\tau_a}, \tag{6}$$

where two relaxation times, τ_0 and τ_a, are employed. The first term in (6) is taken to be that of the base oil, while the second term is associated with the effect of the additive. The characteristic relaxation time of the base oil is very short and normally of no concern in lubrication problems. The relaxation time, τ_a, associated with the interaction between the additive molecules and the base oil is generally taken to be in the range of about 10^{-5}—10^{-3} s. The steady-state shear-flow viscosity associated with (6) is given by

$$\eta = \beta_0 \tau_0 + \beta_a \tau_a . \tag{7}$$

The relative magnitudes of the β_0 and β_a amplitudes in (6) and (7) will be considered later. Form (6) will be shown to separate conveniently the effects of the base oil and the additive. Although we loosely speak of the second term in (6) as being due to the additive, it, of course, is actually a term of interactive effect between the additive and the base oil.

Under simple shear-flow conditions, the stress relations (5), together with the relaxation function (6), take the form

$$
\begin{aligned}
\sigma_{xy} &= (\beta_0\tau_0 + \beta_a\tau_b)\,\frac{\partial u}{\partial y},\\
\sigma_{xx} &= -p - (\beta_0\tau_0^2 + \beta_a\tau_a^2)\left(\frac{\partial u}{\partial y}\right)^2,\\
\sigma_{yy} &= -p - 3\,(\beta_0\tau_0^2 + \beta_a\tau_a^2)\left(\frac{\partial u}{\partial y}\right)^2,
\end{aligned}
\tag{8}
$$

where a coordinate convention (as in the slider bearing of Fig. 1) is used, with u being the velocity in the x direction. The steady-state-stress constitutive equations (8) will be applied to model the flow in the slider bearing of Fig. 1. Thus an assumption of quasi-steady flow is implied, with all transient effects in the non-Newtonian fluid being neglected.

Under ordinary conditions, normal stress effects are of no importance for base oils in lubrication because of the extreme smallness of the relaxation time τ_0. Thus the $\beta_0\tau_0^2$ terms in (8) will be neglected compared with the $\beta_a\tau_a^2$ terms, which involve a much longer relaxation time. A similar reduction in the shear stress term in (8) cannot be made, because the additive is known to have a strong effect on viscosity, and relaxation times enter viscosity to a different order than they enter the normal stress terms. The reduction leaves (8) in the form

$$
\begin{aligned}
\sigma_{xy} &= (\beta_0\tau_0 + \beta_a\tau_a)\,\frac{\partial u}{\partial y},\\
\sigma_{xx} &= -p - \beta_a\tau_a^2\left(\frac{\partial u}{\partial y}\right)^2,\\
\sigma_{yy} &= -p - 3\beta_a\tau_a^2\left(\frac{\partial u}{\partial}\right)^2.
\end{aligned}
\tag{9}
$$

Our objective is to assess the effect of the normal stress terms in the slider-bearing configuration. We proceed by the usual analysis of slider-bearing lubrication. Specifically, following the lubrication approximation, we consider only the momentum equation,

$$
\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} = 0, \tag{10}
$$

taking the other equation to be of higher order. Substituting (9) into (10)

gives

$$\eta \frac{\partial^2 u}{\partial y^2} - \beta_a \tau_a^2 \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)^2 = \frac{\partial p}{\partial x}, \tag{11}$$

where η is given by (7). Let the velocity u be given by

$$u = u_0 + u_1 y + u_2 y^2 + \ldots \tag{12}$$

where truncation will be taken at the explicit level shown and (11) becomes

$$2\,\eta u_2 - \beta_a \tau_a^2 \frac{\partial}{\partial x}[u_1 + 2\,u_2 y]^2 = \frac{\partial p}{\partial x}. \tag{13}$$

The boundary conditions are of the type $u = U$ at $y = 0$, and $u = 0$ at $y = h$. Satisfying these conditions as well as (13), to lowest order consistent with the lubrication approximation, gives u as

$$u = \left(\frac{-U}{h} - \frac{h}{2\eta}\frac{\partial p}{\partial x}\right)y + \frac{1}{2\,\eta}\frac{\partial p}{\partial x}\;y^2 + U. \tag{14}$$

We see that, as a natural consequence of the lubrication approximation, the non-Newtonian effects do not appear in (14).

The continuity equation is

$$\partial q_x/\partial x = 0. \tag{15}$$

where

$$q_x = \int_0^h u\,\mathrm{d}y, \tag{16}$$

and (15) takes the form

$$6U\eta \frac{\partial h}{\partial x} = \frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right). \tag{17}$$

where $\partial p/\partial x$ is given by

$$\frac{\partial p}{\partial x} = 6\eta\left(\frac{U}{h^2} - \frac{2q_x}{h^3}\right). \tag{18}$$

We will use the conventions in Fig. 1, taken as in the derivation of Batchelor [11], with

$$h = h_1 - \alpha x. \tag{19}$$

Integrating (17) gives

$$p - p_0 = \frac{6\eta}{\alpha}\left[U\left(\frac{1}{h} - \frac{1}{h_1}\right) - q_x\left(\frac{1}{h^2} - \frac{1}{h_1^2}\right)\right]. \tag{20}$$

At $h = h_2$, we take $p = p_0$ such that the pressure is the same at each end of

the slider bearing. This procedure gives

$$q_x = U\left(\frac{h_1 h_2}{h_1 + h_2}\right). \tag{21}$$

leaving (20) as

$$p - p_0 = \left(\frac{6\eta U}{\alpha}\right)\frac{(h_1 - h)(h - h_2)}{h^2(h_1 + h_2)}. \tag{22}$$

We shall have use for the integral of (22) which is

$$\int_0^l (p - p_0)\,\mathrm{d}x = \frac{6\eta U}{\alpha^2}\left[\ln\frac{h_1}{h_2} - 2\left(\frac{h_1 - h_2}{h_1 + h_2}\right)\right]. \tag{23}$$

Our next objective is to determine the total load that can be supported by the bearing. The stress integral is decomposed into the parts

$$\int_0^l \sigma_{yy}\,\mathrm{d}x = \int_0^l (-p)\,\mathrm{d}x + \int_0^l \hat{\sigma}_{yy|y=0}\,\mathrm{d}x\,, \tag{24}$$

where the first term on the right-hand side of (24) is given by (23); and from (9), the last term has $\hat{\sigma}_{yy|y=0}$ given by

$$\hat{\sigma}_{yy|y=0} = -3\beta_a\tau_a^2\left(\frac{\partial u}{\partial y}\bigg|_{y=0}\right)^2. \tag{25}$$

Using the preceding results to evaluate the derivative in (25) gives

$$\hat{\sigma}_{yy|y=0} = -12\beta_a\tau_a^2\left[-2 + \frac{3h_1h_2}{h(h_1 + h_2)}\right]^2\frac{U^2}{h^2}. \tag{26}$$

When $h = h(x)$, given by (19), is substituted into (26) and then integrated, we obtain

$$\int_0^l \hat{\sigma}_{yy}\,\mathrm{d}x = -\frac{12\beta_a\tau_a^2U^2}{\alpha}\left[-\frac{2(h_1 - h_2)}{h_1h_2} + \frac{3(h_1^3 - h_2^3)}{h_1h_2(h_1 + h_2)^2}\right]. \tag{27}$$

Now, substituting (23) and (27) into (24) and rearranging the algebraic forms gives

$$-\int_0^l \sigma_{yy}\,\mathrm{d}x = p_0 l + \frac{6\eta U}{\alpha^2}\left\{\left[\ln\left(\frac{h_1}{h_2}\right) - 2\left(\frac{h_1/h_2 - 1}{h_1/h_2 + 1}\right)\right] + \right.$$

$$\left. + \frac{2\beta_a\tau_a^2U}{\eta l}\left[\left(\frac{h_2}{h_1}\right)\frac{(h_1/h_2 - 1)^2(h_1^2/h_2^2 - h_1/h_2 + 1)}{(h_1/h_2 + 1)^2}\right]\right\}. \tag{28}$$

This expression gives the total load that can be supported by the bearing.

The first bracketed term in (28) is just due to the wedging effect of the bearing, and it involves only Newtonian-flow characteristics. This term is the usual Reynolds solution for the lubrication effect with a Newtonian fluid. The last term in (28) is the non-Newtonian effect resulting from the normal stresses. It is the relative size of these two terms that we wish to evaluate explicitly.

Before considering examples of particular bearing configurations, we should examine the general characteristics of the non-Newtonian term in (28). Note first that this non-Newtonian term does not involve any materials parameters that are independent of those involved in the viscosity coefficient η, given by (7). Thus, as we shall see, we can evaluate the size of the non-Newtonian term without appealing to independent measurements of the normal stress, measurements which are particularly difficult to determine.

For first interpretive purposes, take the ratio of h_1/h_2 to be in the range of 2 to 5. Then the order of the non-Newtonian term inside the braces in (28) can be written as

$$0\ (\text{non-Newtonian term}) = \beta_a \tau_a^2 U/\eta l\,. \tag{29}$$

Because in (7) the additive makes a significant contribution to the viscosity so that $\beta_0 \tau_0$ and $\beta_a \tau_a$ are of the same order, then (29) reduces to

$$0\ (\text{non-Newtonian term}) = \tau_a U/l\,; \tag{30}$$

Thus, we see that the strength of the non-Newtonian effect relative to the Newtonian effect is not governed by the Deborah number, which would be expressed as $\tau_a U/h$. Of course, the absolute (not relative) magnitude of the non-Newtonian term is directly related to the magnitude of the normal stress effect.

3. Example

The solution for the bearing load (28) applies only to the slider bearing of Fig. 1. However, we take a more general point of view and consider the solution not only as a direct model for a tilted pad bearing but also as an indirect example for a journal bearing. We take as a typical example the set of data $h_1/h_2 = 2$, and $l = 0.05$ m. With these data, (28) becomes

$$\int_0^l -\sigma_{yy}\, dx = p_0 l + 0.16\, \frac{\eta U}{\alpha^2} \left\{ 1 + 251.8\, \frac{\beta_a \tau_a^2 U}{\eta} \right\}. \tag{31}$$

Henceforth, we shall only be concerned with the term in braces in (31) as representing the relative effect of the Newtonian and the non-Newtonian contributions to the bearing load. Designate the ratio of the non-Newtonian to the Newtonian load by λ, then from (7) and (31)

$$\lambda = \frac{\text{non-Newtonian load}}{\text{Newtonian load}} = 251.8 \left(\frac{\beta_a \tau_a}{\beta_0 \tau_0 + \beta_a \tau_a} \right) \tau_a U\,. \tag{32}$$

We take $\lambda = 0.2$ as representing the practical case of the threshold of a significant non-Newtonian effect, such that the non-Newtonian effect supports 17% of the total bearing load. At the practical speed of $U = 20$ m/s, eqn. (32) then gives

$$\tau_a \left(\frac{1}{1 + \beta_0\tau_0/\beta_a\tau_a}\right) = 3.971 \times 10^{-5}\,. \tag{33}$$

Alternatively, at $\lambda = 0.2$ and $\beta_a\tau_a/\beta_0\tau_0 = 2$, then (32) gives

$$U = (1.191 \times 10^{-3})/\tau_a \tag{34}$$

This latter case corresponds to the additive effect that causes the viscosity to be increased by a factor of three.

4. Discussion

The results from (33) and (34) are displayed in Figs. 2 and 3. We will discuss these explicit results and then the pertinent qualification and limitations upon them. In Fig. 2, we have a cross-plot of the bearing speed versus the relaxation time (of the additive) that increases the viscosity of the oil by a factor of three. It is commonly agreed that the addition of commercial additives to base oils increases the viscosity by a factor of as much as two or three. The relaxation times of additives in oil solution have been reported

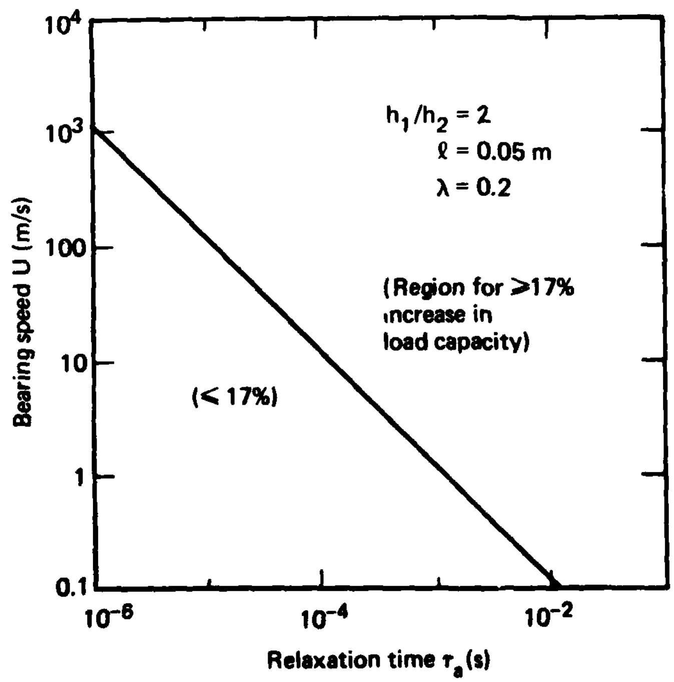

Fig. 2. Bearing speed as a function of relaxation time of the additive at $\beta_a\tau_a/\beta_0\tau_0 = 2$.

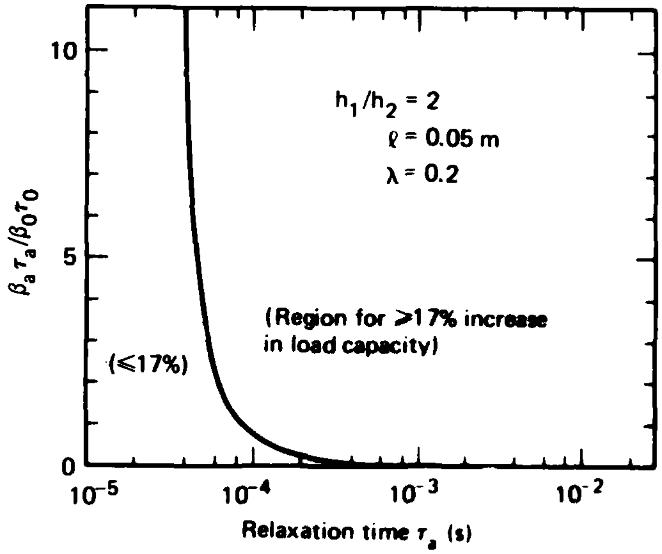

Fig. 3. Ratio of viscosity contribution of additive ($\beta_a \tau_a$) to viscosity contribution of base oil ($\beta_0 \tau_0$) as a function of relaxation time of the additive for $U = 20$ m/s.

over the range 10^{-5} to 10^{-3} s. From Fig. 2, we can determine, at a given relaxation time, the bearing speed at which a significant portion of the load is carried by the non-Newtonian effect. For example, at $\tau_a = 10^{-4}$ s, the bearing speed must be at least 12 m/s for the non-Newtonian effect to be significant (as specified by $\lambda = 0.2$). Interestingly, this speed is in the range of practical bearing operation. At lower speeds, the non-Newtonian effect is predicted to be negligible. For speeds greater than 12 m/s, the ratio of the non-Newtonian load to the Newtonian load increases proportionally with velocity.

In Fig. 3, at a fixed bearing speed of 20 m/s, we plot the relaxation time versus the contribution of the additive to the viscosity, to obtain a threshold of significant load support by the non-Newtonian effect. For the practical range of $0.5 \leq \beta_a \tau_a / \beta_0 \tau_0 \leq 5$, Fig. 3 shows that the relaxation time must be about 10^{-4} s. If the relaxation time is significantly less than this level, then the non-Newtonian effect is negligible. Alternatively, at $\tau_a = 10^{-4}$ s, the additive must be inserted in sufficient quantity to increase the viscosity by at least a factor of 1.66 for the non-Newtonian load term to be significant (as specified by $\lambda = 0.2$).

We see from the preceding examples that the present theory predicts a non-Newtonian effect that is at a threshold significance in the range of practical bearing and fluid characteristics. These characteristics include the geometry of the bearing, the speed, the relaxation time of the additive in solution, and the contribution of the additive to the total viscosity of the fluid. The examples show that the load-bearing capability of the non-Newtonian effect

is significant for certain practical ranges of these parameters but is insignificant for others. In general, the effect must be evaluated explicitly from the result (28) to determine its possible significance. The added effect of shear thinning is discussed below.

It is an important feature of the present method that only the relaxation time of the additive and its effect on the viscosity are needed to evaluate the load capacity of the bearing. This is in contrast to many second-order theories where an independent normal stress coefficient is needed to evaluate the magnitude of the non-Newtonian effect. Of course, the present use of the Coleman—Noll second-order theory limits the range of applicable rates, as does any second-order theory. However, in the present application, only the relative effect of Newtonian and non-Newtonian sources is sought. As seen from (32), this relative effect involves the ratio

$$\frac{\beta_a \tau_a}{\beta_0 \tau_0 + \beta_a \tau_a} \tau_a .$$

Even though the second-order theory does not model shear and normal stress thinning effects, the above term shows that only the ratio of the corresponding material-property coefficients and the relaxation time are involved. If equivalent-magnitude thinning effects were to occur in shear and normal stress terms, then the results would have a validity beyond the range of the usual second-order theory. Typically, however, the normal stresses show a stronger thinning effect than does the shear stress. In fluids where this situation is the case, the present method shows that the effect of the additive would give negligible load support through the normal stress contribution. Thus the thinning effect is of importance to the lubrication applications. Certainly, the present results show the strength of the non-Newtonian effect in the shear-rate range of the inception of non-Newtonian effects, and these results provide a basic solution in that respect. With regard to practical lubrication problems, the preceding numerical examples must be applied with caution for the reasons just mentioned.

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